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REVIEWS

Computational Methods for Turbulent, Transonic, and Viscous Flows. Edited by J. A. Essers. Hemisphere/Springer, 1983, 360 pp. DM 115.

Computational Methods for Fluid Flow. By Roger Peyret and Thomas D. Taylor. Springer, 1983, 358 pp. DM 92.

Computational fluid dynamics, often abbreviated CFD, is a mode of investigation very definitely on the rise these days. Although this may be seen as simply another manifestation of a current, generally enhanced, interest in scientific computation, fluid mechanics has always seemed particularly well suited for 'numerical experimentation': the basic laws governing fluid motion are usually well established, whereas the variety of phenomena accessible within the solution set is largely unknown. Typically the achievable degree of nonlinearity increases with the size and speed of the available computer. And, with the strides in hardware technology taken even during the past decade, regimes of fluid flow of considerable interest are yielding to computation, and results that hardly are accessible by any other method are being obtained.

The successful CFD investigation ideally incorporates a balanced mix of the following ingredients: (1) an awareness of fluid-dynamical problems of scientific interest and/or practical importance; (2) an understanding of the requisite physical laws to be used; (3) an appreciation for numerical analysis issues leading to the choice and verification of an algorithm; (4) a reasonable level of 'craftsmanship' in the implementation; (5) a perceptive analysis and diagnosis of results; and (6) predictions and recommendations for further work both theoretical, experimental and numerical. Many papers and books on CFD unfortunately fall short of this ideal.

The volume edited by Essers is a collection of six articles based on lecture series given by various authors at the von Kármán Institute during the Spring of 1981. The titles of the articles and their authors are:

'Numerical methods for coordinate generation based on a mapping technique' by R. T. Davis;
'Introduction to multi-grid methods for the numerical solution of boundary value problems' by W. Hackbusch;
'Higher level simulations of turbulent flows' by J. H. Ferziger;
'Numerical methods for two- and three-dimensional recirculating flows' by R. I. Issa;
'The computation of transonic potential flow' by T. J. Baker;
'The calculation of steady transonic flow by Euler equations with relaxation methods' by E. Dick.

Of these, the two longest by Ferziger and Baker come closest to the 'ideal' outlined previously, and hence are the most interesting. Both give comprehensive overviews of their subject matter and seem very useful for an introduction. The problem with turbulence calculations, of course, is that even the mightiest computers are barely able to handle Reynolds numbers of genuine interest, and the results of simulating model equations are somehow never entirely convincing. The computer is a forgiving piece of equipment. Unlike real fluids in the laboratory it will happily generate flows that are physically unrealizable.

Ferziger takes a largely pragmatic view, as do many practitioners of turbulence modelling, but then is forced to rely on laboratory experiments for 'verification' (sometimes, less charitably, this is called 'post-diction') and adjustable parameters. The article does, however, also contain some discussion of what is called 'full simulation', which means that one uses the full Navier–Stokes equations. Except for an index, no attempt is made in this volume to achieve coherence between the various contributions, and, except for being physically bound together between the same two covers, the articles are completely independent. This could just as well have been a volume in a review journal.

If one looks at the actual book literature on CFD, the volume by Peyret & Taylor being a recent addition in the Springer Series in Computational Physics, one discovers not inconsiderable bias towards item (3) on my list: methods. Books on CFD tend to be written much like the 'mathematical methods' texts that preceded them. They are often scholarly and frequently useful, but carry connotations of only being 'part of the story'. Peyret & Taylor's volume is no exception. While probably valuable in a graduate course, it can hardly stand alone. The overview of methods given is extensive, although of varying depth (the comprehensive title notwithstanding), e.g. free-surface flows are not discussed, but the discussion of results is too light. There is little comparison of various different numerical approaches to a given physical problem. A case in point: section I.4, entitled 'Relationship between Numerical Approaches', is just four pages long! The amount of numerical analysis seems about right for people who want to *do* (as opposed to *theorize about*) numerical calculations. There are no 'convergence theorems' here, no Sobolev spaces etc. Unfortunately, possibly in an attempt to please a wider audience, this book tends not to voice definite opinions about particular methodologies, thus leaving the reader in search of a coherent point of view somewhat adrift. Nevertheless, in spite of these shortcomings, which the book shares with scores of others in the genre, what is done in this volume is done competently, and I suspect that the book can and will grow through future editions to become a valued reference work in numerical methodology.

In summary, the collection edited by Essers is essentially a set of review articles written in journal format with the more comprehensive ones being the more interesting. The volume should be acquired by individuals only on the basis of interest in one or more of the topics taken up, but should be acquired by libraries because it contains some good articles that presumably are not available elsewhere. The text by Peyret & Taylor would be useful as one of several for a graduate course in CFD, and for reference and study by researchers in the field. It is a methods book with limited coverage and needs supplementing both with regard to methods not covered and particularly with regard to applications. It is not a theoretical numerical-analysis study.

Nominally CFD spans a wide spectrum of topics ranging from practically motivated calculations in specific applications to the elucidation of basic physical phenomena and mathematical mechanisms within the equations for fluid flow. It is without doubt primarily in this latter mode that most of the scientific excitement is to be found. Numerical experiments that turn up such objects as solitons or strange attractors are typically done on very idealized systems! There are many interesting issues here (some of them quite deep, it would appear, including questions of methodology), which are to my knowledge not treated at all in the current book literature. Such items are possibly closer to the heart of the subject than much of the routine material treated in the methods books one currently sees. As the field of CFD matures one would hope to see reflected in the book literature a shift in subtlety and depth away from what has to be an immediate response to pressures of the marketplace towards more basic scientific issues.

HASSAN AREF

Regular and Stochastic Motion. By A. J. LICHTENBERG and M. A. LIEBERMAN. Springer, 1983. Applied Mathematical Sciences Series no. 38. 499 pp. DM 108 or US $44.60.

At first sight, this book, despite its title, which might suggest analogy with laminar and turbulent flow, has apparently little direct contact with fluid mechanics. The authors write in the Preface that 'the main emphasis is on intrinsic stochasticity in Hamiltonian systems, where the stochastic motion is generated by the dynamics itself and not by external noise'. And, although a final chapter of some sixty pages is devoted to 'Dissipative Systems' which takes the Lorenz system as a prime example, and which makes ritual reference to the problem of 'the Transition to Turbulence' in a concluding section, it is the classical Hamiltonian systems of nonlinear oscillation of two and more degrees of freedom which provide the starting point for the book, and the base from which the central chapters are constructed.

This should not necessarily diminish the potential interest of the subject from the fluid-dynamical point of view. As pointed out recently by H. Aref in this *Journal* [**143**, 1–21], the familiar equations for particle trajectories in any two-dimensional incompressible flow, viz

$$\dot{x} = \frac{\partial \psi}{\partial y}, \qquad \dot{y} = -\frac{\partial \psi}{\partial x}, \tag{1}$$

where $\psi(x, y, t)$ is the stream function of the flow, constitute a one-degree-of-freedom Hamiltonian system, with Hamiltonian ψ; if the flow is unsteady (i.e. ψ depends explicitly on t), then the system is non-autonomous, but it is then equivalent to an autonomous Hamiltonian system with two degrees of freedom, and the accumulated knowledge concerning such systems since the time of Poincaré must tell us *something* about the relationship between Lagrangian and Eulerian descriptions of fluid motion, which, as anyone who has given a first course in Fluid Mechanics would readily admit, is at the heart of the subject.

And there is no doubt that there have been dramatic advances in our understanding of Hamiltonian dynamics over the last thirty years or so, since Kolmogorov (1954) conjectured that at least *some* two-dimensional integral surfaces in the four-dimensional phase space of a two-degrees-of-freedom Hamiltonian system, perturbed about an integrable state, would survive the perturbation, a conjecture validated by the work of Kolmogorov's student V. I. Arnold (1961–1963: *Sov. Math. Dokl.* **2**, 501; **3**, 136) and independently by J. Moser (1962: *Nachr. Akad. Wiss. Gött. Math.-Phys. Kl.* **1**). These surviving surfaces are the famous 'KAM tori', and they survive provided that the associated winding number q (i.e. the ratio of the natural frequencies in the unperturbed system) is 'sufficiently irrational' in the sense that

$$|q - m/n| > cn^{-\frac{5}{2}} \tag{2}$$

for all integers m, n, and for some constant c related to the magnitude of the perturbation. If q is rational, or so nearly rational that the inequality (2) is violated for some choice of m, n, then the KAM curves (in any plane of section) break up into island chains of extraordinary complexity, exhibiting structure on all lengthscales and associated self-similarity that lends itself to a type of renormalization-group analysis. This break-up of KAM surfaces signals the transition from regular to chaotic orbits in phase space, like the transition from laminar to turbulent flow in unstable fluid systems.

Of course, the equations (1) do not contain any of the real dynamics of fluid systems; they are merely kinematic in content. Nevertheless they are relevant to the diffusion of any convected scalar field which (in the absence of molecular diffusion) will be limited by the existence of KAM surfaces which act like impenetrable barriers, a phenomenon that may be observed in the computations of Aref (*loc. cit.*).

But the relevance to fluid mechanics of Hamiltonian dynamics is not limited to diffusion problems: as recognized by Poincaré himself, a system of N point vortices κ_i at positions $\boldsymbol{x}_i = (x_i, y_i)$ evolves as an autonomous Hamiltonian system of N degrees of freedom, with Hamiltonian

$$H = -\frac{1}{4\pi} \sum_{i \neq j} \kappa_i \kappa_j \ln |\boldsymbol{x}_i - \boldsymbol{x}_j|.$$

There are four integral invariants, and the system is (in general) non-integrable if $N \geqslant 4$; so the phase trajectories (and, in this case, this means the paths of the vortices themselves) are in general chaotic. An understanding of Hamiltonian systems of at least four degrees of freedom should provide insights into this type of problem. The phenomenon of 'Arnold diffusion' – a sort of diffusive meandering of trajectories through a network of KAM surfaces in the phase space – may play an important role in this context.

Lichtenberg and Lieberman provide a masterful survey of these topics. They start with a general overview in Chapter 1, which conveys the flavour of the subject without indigestible detail. Chapter 2 treats canonical perturbation theory and the root problem of small denominators arising through resonant interactions between different degrees of freedom of a system. It is the divergences associated with these small denominators that leads to the destruction of KAM surfaces. The nonlinear oscillator with two degrees of freedom can be reduced to the problem of an iterated area-preserving mapping of a plane onto itself, and this problem is treated in Chapter 3. Here the fascinating generic structure of such systems is revealed, with the hierarchies of elliptic islands separated by hyperbolic fixed points, which is where the chaos tends to be localized. Chapter 4 treats the transition from regular to chaotic behaviour as the perturbation parameter k of a dynamical system passes through a critical value k_c; this leans heavily on the seminal work of J. Greene (1979: *J. Math. Phys.* **20**, 1183) who succeeded in establishing the relationship between the break-up of a KAM torus of winding number q and the loss of stability of the periodic orbits associated with a sequence of rationals $\{q_i\}$ approximating q. When $k = k_c$ the KAM surface is apparently crinkled on all lengthscales (Shenker & Kadanoff 1982; *J. Stat. Phys.* **4**, 631) and its spectrum exhibits self-similarity under successive expansion of scale. The intimate interaction between analytical and numerical work is a characteristic of these papers which is well conveyed by L. & L.'s treatment.

Chapter 5 treats diffusion in regions of the phase space in which the motion is stochastic, and Chapter 6 extends the treatment to three or more degrees of freedom, where Arnold diffusion is a new feature. In a stochastic region, particle trajectories diverge exponentially (in contrast to the linear divergence in regular regions), and an analogy with turbulent diffusion may be drawn. These chapters treat such concepts as Liapunov exponents and Kolmogorov entropy, and the question of whether the system behaviour can be described in terms of a diffusion equation in 'action space'.

It will be evident from this very brief description that a fascinating range of topics is covered in the book, and that much may be of potential, if not actual, interest to fluid dynamicists. The book is beautifully illustrated with many varied examples of systems exhibiting regular and chaotic behaviour and the transition between the two, and, at a superficial level, one can gain a good impression of the current state of development of the subject. If one wants to do more than this, and gain understanding in depth, the book will not be sufficient on its own: the treatment of fundamental topics will be found too brief and compressed by anyone who is not already thoroughly familiar with the material. If one is prepared to consult the key original papers (such as those referred to in this review), then the book becomes more easily comprehensible; but it will undoubtedly be hard reading for anyone who does not have access to these earlier papers or to more recent reviews.

Take for example the discussion of the behaviour of trajectories near a hyperbolic singular point of an area-preserving mapping, as given on p. 170. The authors define the incoming and outgoing trajectories H^+ and H^- and then baldly assert that "the H^- curve leaving one hyperbolic point generically intersects the H^+ curve arriving at the neighbouring...hyperbolic point". Well, this may be true, but no justification is given, and one would undoubtedly have to search back to earlier references to understand *why* it is true. The fact that one intersection implies an infinity of others in the neighbourhood of the hyperbolic point makes this a rather crucial assertion as regards the developments of subsequent chapters, and one which merits closer scrutiny than it gets here.

Likewise, one may criticize the discussion of the KAM theorem as given in section 3.2. The discussion on pp. 162–164 is extraordinarily difficult to follow, and that is unfortunate, since it is central to the whole book. The theorem is of course a difficult one, and no truly simple proof of it has yet been given. However, once the problem is stated in terms of two-dimensional mappings, it is purely geometric in character (as the 1962 paper of Moser made clear), and a purely geometric discussion avoiding dynamical terminology (i.e. such words as action, frequency, Hamiltonian, generating function) is then desirable in the interests of clarity and simplicity. The discussion of this text is conducted at a level of dynamical maturity that is seldom found in 'non-expert' researchers, far less in the graduate students at whom the book is partly directed.

Despite these slight reservations, I have a very favourable impression of the book as a whole, which contains a vast range of fascinating material that is likely to become of increasing relevance in fluid-mechanical contexts. The authors are to be congratulated in having provided an authoritative account of topics which are of great current interest in a variety of fields.

H. K. MOFFATT